Imprecise-Chance Markov Chains

Kouroush Jenab, Ph.D
Department of Mechanical and Aerospace Engineering,
University of Ottawa
Ottawa, Ontario, Canada
Kouroush.jenab@sympatico.ca

B.S. Dhillon, Ph.D, P.Eng
Department of Mechanical and Aerospace Engineering,
University of Ottawa
Ottawa, Ontario, Canada
dhillon@genie.uottawa.ca

Abstract

This paper presents the use of an imprecise-chance Markov chains with stochastic transition rate in the case of part parameter change using a fuzzy flow-rate equation system associated with interval chance. The application of the method is demonstrated by solving the cooling unit Markov model for the switching box used in telecommunication.

1.Introduction

Markov chains are useful for systems with dependent failure and repair modes when components within the system are independent, and when multi-state systems with common failures require analysis. Due to uncertain and time-varying input reliability data, the use of fuzzy sets in Markov chains has been considered in some publications that can be classified as the use of fuzzy probability distribution [References: Jenab K. et al , Kurse, R. et al] or the fuzzy possibility distribution [Reference: Sankaranarayanan, V. et al] in the transition matrix. To calculate the steady state distribution of Markov chains, the flow-rate equation ($\sum_{i=1}^{n} t_{i,k}.P(i) - P(k)\sum_{j=1}^{n} t_{k,j} = 0$ for k=1,2,, n (Eq.1)) can be reduced to a simple closed form. Though, until now no attention has been paid to fuzzy flow-rate equation system associated with interval chance. Thus, Our objective is to present the fuzzy flow-rate equations to deal with time varying input for the states of Markov chains. In addition, because the equations are associated with interval chance to represent uncertain input data, the method is called Imprecise-Chance Markov chains.

1.1Notation

$\mathrm{P}\tilde{\mathrm{r}}\,ob(\,)$	Fuzzy probability function
α_k	Probability that flow-rate equation lies within specified interval (i.e., $b_k \pm \Delta b_k$)
$\Delta\alpha_k$	Specified interval for probability that flow-rate equation lies within specified interval (i.e., $b_k \pm \Delta b_k$)
n	Total number of states
$t_{i,j}$	Transition rate from state i to state j that is a random variable with normal probability distribution
$\mu_{i,j}$	Mean of transition rate from state i to j
$\sigma_{i,j}$	Standard deviation of transition rate from state i to state j
P(i)	Probability of being in state i
μ_k	Mean of rate of entry to minus rate of exit from state k
σ_k	Standard deviation of rate of entry to minus rate of exit from state k
b_k	Rate of entry to minus rate of exit from state k in flow-rate, which is assumed equal to zero
Δb_k	Specified interval for flow-rate equation (tolerance)
$\Phi(\,)$	Cumulative distribution function of normal distribution
$\mu_{\sum_{i=1}^{n} P(i)}$	Possibility function of $\sum_{i=1}^{n} P(i)$
Δ_p	Specified chance interval for $\sum_{i=1}^{n} P(i)$

2.Problem description

Both heat and cold accelerate chemical and physical deterioration. The physical properties of components are modified by temperature and the component failure rate doubles for every 10°C rise in temperature. In electronic design, because of aging effect and stress (e.g., dynamic, electrical, thermal), the failure rate of component changes with time. Moreover, failure rates calculated in accordance with MIL-HDBK-217 do not normally address circuit failures caused by stress and aging effects, which result in fuzzy form of Equation (1). This form can be associated with specified chance as follows:

$\Pr\tilde{o}b(\sum_{i=1}^{n} t_{i,k}.P(i) - P(k)\sum_{j=1}^{n} t_{k,j} \cong 0) \gtrapprox \alpha_k$ for k=1,2,…, n (Eq.2). The specified chance represents possibility that the flow-rate equation lies within the specified interval. Moreover, due to uncertain and time varing component failure rates, the chance should be expressed using the interval method. The imprecise-chance flow-rate equation comprises two fuzzy parts. In a steady state situation, the first part is the flow-rate equation, which is the rate of entry into the state 'k' minus the rate of exit from state 'k'. It lies within specified interval (e.g., [$-\Delta b_k$,$+\Delta b_k$]) with respect to aging and stress effects on component failure rate. The second part is the interval chance (i.e.,($\alpha - \Delta\alpha$, α_k) that represents probability that the flow-rate equation lies within specified interval $\Delta\alpha$. Thus, we have the fuzzy equation system made up of 'n' imprecise-chance flow-rate equations that cannot be solved using the matrix technique. In order to convert and solve such an equation system (i.e., Equation 2 and $\sum_{i=1}^{n} P(i)$ =1 (Eq.3)), we assume that the transition rates are random variable with normal probability distribution.

3.Fuzzy Markov Chains with imprecise-chance flow-rate equations

In order to solve the imprecise-chance flow-rate equations (i.e., Eq.2), it needs to be converted to non-fuzzy form in steps as follows:

3.1 Step 1: Conversion of the imprecise-chance flow-rate equations (Equation (2))

Since $t_{i,k}$ is an independently and normally distributed random variable with mean $\mu_{i,k}$ and standard deviation $\sigma_{i,k}$, Equation (4) yields $\Pr\tilde{o}b(\frac{y_k - \mu_k}{\sigma_k} \cong \frac{b_k - \mu_k}{\sigma_k}) \gtrapprox \alpha_k$ for k=1,2,…, n (Eq.4). Where $b_k = 0$,and $y_k = \sum_{i=1}^{n} P(i).t_{i,k} - \sum_{j=1}^{n} P(k).t_{k,j}$ (Eq.5), $\mu_k = \sum_{i=1}^{n} P(i).\mu_{i,k} - \sum_{j=1}^{n} P(k).\mu_{k,j}$ (Eq.6), $\sigma_k^2 = \sum_{i=1}^{n} P(i)^2.\sigma_{i,k}^2 + \sum_{j=1}^{n} P(k)^2.\sigma_{k,j}^2$ (Eq.7). Thus, Equation (4) leads to $2\Delta_{b_k} - \Phi^{-1}(1-\alpha_k - \Delta_{\alpha_k}).\sigma_k \geq 0$ for k=1,2,…,n (Eq.8).

3.2 Step 2: Determine possibility of probability interval for Equation (8)

Based on Figure 1, the possibility of Equation (3) is expressed by Equation (9). To solve the converted flow-rate equation system, the possibility is considered as objective that need to be maximized subject to the converted fuzzy flow-rate equation system to non-fuzzy.

$$\mu_{\sum_{i=1}^{n} P(i)} = \begin{cases} A: 0 & \text{for } \sum_{i=1}^{n} P(i) \le 1-\Delta_p \text{ .or. } \sum_{i=1}^{n} P(i) \ge 1+\Delta_p \\ B: \dfrac{\sum_{i=1}^{n} P(i) - (1-\Delta_p)}{\Delta_p} & \text{for } 1-\Delta_p < \sum_{i=1}^{n} P(i) \le 1 \\ C: \dfrac{1+\Delta_p - \sum_{i=1}^{n} P(i)}{\Delta_p} & \text{for } 1 \le \sum_{i=1}^{n} P(i) < 1+\Delta_p \end{cases} \qquad (Eq.9)$$

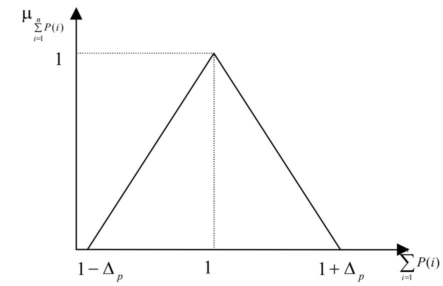

Figure 1: Possibility function of $\sum_{i=1}^{n} P(i)$

3.3 Step 3: Development of Mathematical model

To solve the non-fuzzy equation system composed of Equations (8) and (9), mathematical modelling is proposed.

Objective: Max : G (Eq.10)

Subject to: $2\Delta_{b_k} - \Phi^{-1}(1-\alpha_k - \Delta_{\alpha_k}).\sigma_k \ge 0$ for k=1,2,….,n (Eq.11), $G \le \dfrac{1+\Delta_p - \sum_{i=1}^{n} P(i)}{\Delta_p}$ (Eq.12), $G \le \dfrac{\sum_{i=1}^{n} P(i) - 1 + \Delta_p}{\Delta_p}$ (Eq.13), $1-\Delta_p \le \sum_{i=1}^{n} P(i) \le 1+\Delta_p$ (Eq.14), $P(i) \ge 0$ for i=1,2,…,n (Eq.15), and $G \ge 0$ (Eq.16).

Where $\sigma_k = \sqrt{\sum_{i=1}^{n} P(i)^2.\sigma_{i,k}^2 + \sum_{j=1}^{n} P(k)^2.\sigma_{k,j}^2}$ and $G = \mu_{\sum_{i=1}^{n} P(i)}$.

Using software packages such as LINGO for solving the model, the maximum possibility is obtained, which results in the state probabilities.

4.Examples

Figure 2 shows a Markov model of a switching box that requires airflow equal to 2.33*(power dissipation/air temperature rise). The cooling unit composed of two fans configured in dynamic redundancy is used to maintain the airflow. Each fan comprises mechanical parts, connectors, wire, and a power circuit. Table 1 presents normally distributed transition rates, intervals for flow-rate equations (tolerances), flow-rate equation chances, and intervals for the chances.

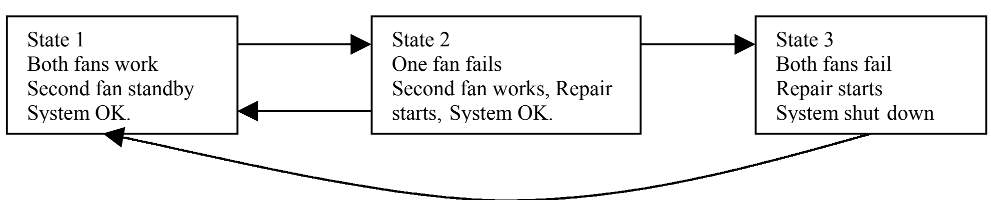

Figure 2: Cooling unit Markov model

By implementing steps 1 to 3, the developed mathematical model is solved by LINGO. Table 2 presents the result indicating that the maximum possibility is 0.9999995. Since cooling is functioning adequately in the states 1 and 2, the probability of system being up is equal to 0.999. More specifically it means that the cooling unit has a down time of 8.8 hours in a year.

Table 1 : Input parameters of cooling unit Markov model

State K	Transition rate (entry) $t_{i,k} \sim N(\mu_{i,k}, \sigma_{i,k})$	Transition rate(exit) $t_{k,j} \sim N(\mu_{k,j}, \sigma_{k,j})$	Interval for equation $(b_k \pm \Delta b_k)$	Chance-interval of equation $(\alpha - \Delta\alpha, \alpha_k)$	Specified interval for sum of probabilities (Δ_p)
1	$t_{2,k} \sim N(5,0)$ $t_{3,k} \sim N(5,0)$	$t_{k,2} \sim N(10,0)$	0 ± 5	(0.95,0.97)	0.05
2	$t_{1,k} \sim N(10,0)$	$t_{k,3} \sim N(10,0)$	0 ± 10	(0.95,0.97)	0.05
3	$t_{2,k} \sim N(10,0)$	$t_{k,1} \sim N(5,0)$	0 ± 7	(0.95,0.97)	0.05

Table 2 : Result of mathematical model for Markov Model

Possibility value: 0.9999995	
State No. and Description	**Probability of being in state**
1-One fan works; the other is standby: system ok	0.148
2-One fan works; the other fails: system ok	0.851
3-Both fans fail: system down	0.001

5.Conclusion

Both heat and cold accelerate chemical and physical deterioration. The physical properties of components are altered by temperature and the component failure rate doubles for every 10°C rise in temperature. In electronic design, because of aging effect and stress (e.g., dynamic, electrical, thermal), the failure rate of component changes with time. Moreover, failure rates calculated in accordance with MIL-HDBK-217 do not normally address circuit failures caused by stress and aging effects, which result in a fuzzy flow-rate equation system that can be associated with specified chance. Thus, to deal with such situtaion, we propose the fuzzy flow-rate equation system. The specified chance represents the possibility that the flow-rate equation lies within a specified interval. The interval may vary for the states because of aging effects and stress. Moreover, due to uncertain and time varing component failure rate, the chance is expressed by the interval method. This approach presents the flow-rate equation system for Markov models with fuzzy operand associated with a specified interval chance. The result is a so-called imprecise-chance equation system. To solve the imprecise-chance flow-rate equation system, we proposes the use of mathematical model obtained using developed approach that converts the equations to non-fuzzy mathematical models that can be solved by software packages like LINGO. The model maximizes the possibility of obtaining the sum of all probabilities, which results in the state probabilites, availability, and down time of system.

References

Jenab, K. Dhillon, B.S. (2000). Switching network analysis via fuzzy Markov modelling considering fuzzy FC, FIT's, and MTTR. 44th Annual conference of the Canadian Operation Research Symposium- Toronto. 52.

Kurse, R. Buck-Emden, R. Cordes, R. (1987). Processor Power Considerations- An Application of Fuzzy Markov Chains. Fuzzy Sets and Systems (21), 289-299.

Sankaranarayanan, V. Seshasayee, S. (1992). Fuzzy reliability modelling – linguistic approach. Microelectronic Reliability. 32(9), 1311-1318.